Cost-optimal Power System Extension Under Flow-based Market Coupling and High Shares of Photovoltaics

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***Abstract*—Electricity market models, implemented as dynamic programming problems, have been applied widely to identify possible pathways towards a cost-optimal and low carbon electricity system. However, the joint optimization of generation and transmission remains challenging, mainly due to the fact that different characteristics and rules apply to commercial and physical exchanges of electricity in meshed networks. This paper presents a methodology that allows to optimize power generation and transmission infrastructures jointly through an iterative approach based on power transfer distribution factors (PTDFs). As PTDFs are linear representations of the physical load flow equations, they can be implemented in a linear programming environment suitable for large scale problems such as the European power system. The algorithm iteratively updates PTDFs when grid infrastructures are modified due to cost-optimal extension and thus yields an optimal solution with a consistent representation of physical load flows. The method is demonstrated on a simplified three-node model where it is found to be stable and convergent. It is then scaled to the European level in order to find the optimal power system infrastructure development under the prescription of strongly decreasing CO_2 emissions in Europe until 2050 with a specific focus on photovoltaic (PV) power.**

I. INTRODUCTION

Motivated by ambitious emission reduction and renewable energy integration targets, the European power system is expected to undergo substantial changes. Electricity market models, implemented as a dynamic programming problem, have been applied widely to identify possible pathways. However, these models mostly lack an appropriate representation of the physical grid which represents the backbone of today's power system. Specifically, a joint optimization of generation and transmission is difficult, mainly due to the fact that different characteristics and rules apply to commercial and physical exchanges of electricity in meshed networks.

This is specifically true when dealing with an intermeshed alternating current (AC) transmission network as the European power system. According to Kirchhoff's circuit law, multiple paths are taken by the physical flows when settling trades from one point to another via the intermeshed grid (so called loop flows), such that a large number of lines may be impacted.

Many studies have dealt with the problem of transmission system expansion. Comprehensive literature surveys for the general problem of transmission system expansion and corresponding modelling issues are provided in [1], [2]. As stated in [1], the problem comprises economic and engineering considerations, which can easily be confirmed when analysing the corresponding fields of research.

From an engineering perspective, early approaches to transmission system expansion can be found in [3] or [4] that both formulate linear load flow equations in order to find overloaded lines, however only considering snapshots of the future power system. Besides linear programming, later works also deploy various other optimization methods, such as non-linear programming, mixed-integer programming or artificial intelligence methods [5].

The second stream of analysing transmission system extensions is mostly based on economic considerations: In [6] the analytical model uses PTDF in order to integrate loop flows that were previously found to have a significant impact on the efficiency of the market outcome in meshed networks [7]. They assume an invariant PTDF matrix and furthermore do not address social welfare effects. A very similar modelling framework is applied in [8] to analyse an incentive mechanism for transmission expansion with a profit-maximizing transmission system company and a competitive wholesale market based on nodal pricing, and in [9] to specifically analyse the impact of different cost functions.

This paper presents a methodology that couples an electricity market model with a power flow model to jointly optimize both power generation and transmission grid infrastructures under flow-based market coupling using an iterative approach based on power transfer distribution factors (PTDFs). The objective of the proposed method is to find the overall cost-optimal solution for serving electricity to the consumers, and thus to optimize social welfare. PTDFs are linear representations of the load flow equations which can be used to calculate physical active power flows in the power network given market transactions. As such, they can be implemented in a linear programming environment determining the cost-optimal development of power system infrastructures under certain restrictions. However, a set of given PTDFs is only valid as long as certain criteria are met, such as no reactive power flows and no losses. Furthermore the PTDFs change with each change of grid configuration, so we suggest a method whereby the PTDFs are updated and fed back every time the grid is modified. The method is demonstrated on a simplified three-node model where the iterative optimization algorithm is found to be stable and convergent.

The paper then provides an outlook on a large-scale application that is currently being implemented in order to find the optimal power system infrastructure development under the prescription of strongly decreasing CO_2 emissions in Europe until 2050 with a specific focus on photovoltaic (PV) power. The following two main questions shall be answered within this framework:

- What does a cost-optimized European power system (both generation and grid) look like in 2030 (medium term) and 2050 (long term)?
- How does an optimized grid extension help to cost-optimally deploy power from photovoltaic (PV) installations in Europe?

The results of this large-scale application will be published in a separate paper.

The remainder of the paper is structured as follows: Section II presents the methodology developed to jointly optimize power generation and transmission grid infrastructures in an iterative manner based on PTDFs. The algorithm is applied to a simple three node network in Section III. Section IV presents an outlook on the modelling framework of the large-scale application to the European power system with large shares of photovoltaics. Section V concludes.

II. METHODOLOGY

This section is subdivided into two parts. First, starting from the most general formulation of the load flow equations in an intermeshed AC grid, a linear PTDF representation is derived suitable for being integrated in a large scale linear optimization problem. Then, a model is presented focusing on the problem of integrating load flow calculations in an economic optimization framework with the objective to find the cost-optimal grid infrastructure in a multi-node network with different load and generation characteristics.

A. Load flow equations and PTDF representation

As noted in most electrical engineering books (e.g. refer to [10]), the most general form of the network equations in an AC power system can be written as follows:

$$\begin{aligned}
P_i &= U_i \sum_{j\in I} U_j (g_{ij}\cos(\delta_i-\delta_j) + b_{ij}\sin(\delta_i-\delta_j)) \\
Q_i &= U_i \sum_{j\in I} U_j (g_{ij}\sin(\delta_i-\delta_j) - b_{ij}\cos(\delta_i-\delta_j)) \\
P_{ij} &= U_i^2 g_{ij} - U_iU_jg_{ij}\cos(\delta_i-\delta_j) - U_iU_jb_{ij}\sin(\delta_i-\delta_j) \\
Q_{ij} &= -U_i^2(b_{ij}+b_{ij}^{sh}) + U_iU_jb_{ij}\cos(\delta_i-\delta_j) \\
&\quad - U_iU_jg_{ij}\sin(\delta_i-\delta_j)
\end{aligned} \tag{1}$$

In the above equations, P_i and Q_i represent the active and reactive power infeed at node i, whereas P_{ij} and Q_{ij} stand for the active and reactive power flow on line ij connecting node i and j, respectively. I is the set of nodes the network consists of. As can be seen, voltage levels U and phase angles δ of the nodes as well as series conductances g and series susceptances b of the transmission lines are determining active and reactive power flows.

There are two well-known algorithms to solve this set of equations, namely the Gauss-Seidel and the Newton-Raphson methods [10]. These algorithms are capable of dealing with the non-linearities in the above equations. Noticeably, both methods are iterative and need an initial guess for all unknown variables. For the purpose of implementing load flow calculations in a linear optimization environment, as presented in this paper, a linear representation of the above equations has to be found. To this end, the following assumptions can be made:

- All voltages are set to 1 p.u., meaning that there is no voltage drop.
- Reactive power is neglected, i.e. Q_i and Q_{ij} is set to zero.
- Losses are neglected, and line reactance is by far larger than the resistance: $X >> R \approx 0$.
- Voltage angle differences are small, such that $sin(\delta_i - \delta_j) \approx \delta_i - \delta_j$.

By making these assumptions, the AC load flow equations can be simplified to a linear relationship:

$$P_{ij} = b_{ij}(\delta_i-\delta_j) = \frac{x_{ij}}{x_{ij}^2+R_{ij}^2}(\delta_i-\delta_j) \approx \frac{1}{x_{ij}}(\delta_i-\delta_j) \tag{2}$$

According to Kirchoff's power law, the active power injection at bus i is then given by

$$P_i = \sum_{j\in\Omega_i}\frac{1}{x_{ij}}(\delta_i-\delta_j) = (\sum_{j\in\Omega_i}\frac{1}{x_{ij}})\delta_i + \sum_{j\in\Omega_i}(-\frac{\delta_j}{x_{ij}}) \tag{3}$$

with Ω_i being the set of buses adjacent to i. For a system with multiple (N) branches, (3) can be written in matrix form as

$$\boldsymbol{P_{nodal}} = \boldsymbol{B}\cdot\boldsymbol{\Theta} \tag{4}$$

where $\boldsymbol{P_{nodal}}$ is the vector containing the net active power injections P_i, $\boldsymbol{\Theta}$ the vector of phase angles and $\boldsymbol{B}$ is the nodal admittance matrix with the following entries:

$$B_{ij} = -\frac{1}{x_{ij}} \tag{5}$$

$$B_{ii} = \sum_{j\in\Omega_i}\frac{1}{x_{ij}} \tag{6}$$

Due to the fact that $\boldsymbol{B}$ is singular, the row and column belonging to the reference bus is deleted (thus assuming a zero reference angle at this bus). The resulting vectors and matrix are named $\boldsymbol{B'}$, $\boldsymbol{\Theta'}$ and $\boldsymbol{P'_{node}}$. We can now solve (4) for $\boldsymbol{\Theta'}$:

$$\boldsymbol{\Theta'} = \boldsymbol{B'}^{-1} \cdot \boldsymbol{P'_{nodal}} \tag{7}$$

Next, we consider the dependency between the load flow on line ij and the phase angle over the same line according to 2 and find the matrix representation to be:

$$\boldsymbol{P_{branch}} = \boldsymbol{H} \cdot \boldsymbol{\Theta}' \tag{8}$$

with $\boldsymbol{P_{branch}}$ the vector of the net active power flows P_{ij} and $H_{ki} = 1/x_{ij}$, $H_{kj} = -1/x_{ij}$ and $H_{km} = 0$ for $m \neq i, j$ (note that k runs over the branches ij). $\boldsymbol{\Theta'}$ can then be inserted in (8) to give:

$$\boldsymbol{P_{branch}} = \boldsymbol{H} \cdot \boldsymbol{\Theta'} = \boldsymbol{H} \cdot \boldsymbol{B'}^{-1} \cdot \boldsymbol{P'_{nodal}} = \boldsymbol{PTDF} \cdot \boldsymbol{P'_{nodal}} \tag{9}$$

The elements of $\boldsymbol{PTDF}$ are the power transfer distribution factors, constituting the linear relationship between the load flows on the lines and nodal power balances.

In the next step, a market model will be introduced that simulates the dispatch of different power plants in different market regions and thus nodal power balances in a cost-minimizing manner. Power flows can then be calculated using the PTDF approach that was introduced in this section, and an additional restriction ensures that line flows stay below thermal limits. Furthermore, the model will be implemented such that thermal limits (i.e. transmission capacity) can be increased when contributing to the cost-optimal solution.

B. Model for the cost optimal expansion of grid infrastructures

The goal of the study presented in this paper is to determine the cost-optimal extension of AC and DC grid infrastructures. To this end, the above deduced linear power flow representation can be embedded in an electricity market model. Herein, an exogenously given demand shall be supplied at least cost by the various technological options of generation and transmission. Market models are commonly modeled as linear optimization problem which is well suited for most applications, especially when large systems with high technological, spatial and temporal resolution shall be analysed. With the methodology deduced in the previous section, load flow calculations and grid extensions can explicitly be included in such a linear program.

Moreover, the methodology presented in this section is also able to account for possible DC grid extensions. Compared to the AC system, flows on the DC lines are easier to deal with due to the fact that all lines are assumed to be point-to-point connections that are equipped with converter stations. This technical equipment makes it possible to perfectly control the flows on the corresponding line, such that trades can directly be settled via those lines (in other words, trades directly translate into physical flows).

Suppose that the level of demand in market i at time t, D_i^t is an exogenous parameter entering the optimization problem. The power that can be generated in market i at time t by technology a at costs of $c_{i,a}^t$ is denoted by $G_{i,a}^t$. Furthermore, transmission capacities between i and j are denoted in vector-form by $\boldsymbol{P_{max}}$ and can be built up at costs of $\boldsymbol{\lambda}$. All quantities are possibly different with respect to space and time.

Within this framework, the following linear program formalizes the optimization problem for the cost-efficient supply of electricity including generation as well as AC and DC transmission expansion.

$$\min \quad C_{tot} = \sum_{i \in I} \sum_{a \in A} \sum_{t \in T} G_{i,a,t} c_{i,a,t} + \boldsymbol{\lambda}^{AC} \cdot \boldsymbol{P}_{max}^{AC} + \boldsymbol{\lambda}^{DC} \cdot \boldsymbol{P}_{max}^{DC} \tag{10}$$

s.t.

$$\sum_{a \in A} G_{i,a,t} + \sum_{j \in J} T_{j,i,t} = D_{i,t} \tag{11}$$

$$T_{i,j} = T_{i,j}^{AC} + T_{i,j}^{DC} \tag{12}$$

$$\boldsymbol{P}^{AC} = \boldsymbol{PTDF} \cdot \boldsymbol{T}^{AC} \tag{13}$$

$$\boldsymbol{P}^{DC} = \boldsymbol{T}^{DC} \tag{14}$$

$$-\boldsymbol{P}_{max}^{AC} \leq \boldsymbol{P}^{AC} \leq \boldsymbol{P}_{max}^{AC} \tag{15}$$

$$-\boldsymbol{P}_{max}^{DC} \leq \boldsymbol{P}^{DC} \leq \boldsymbol{P}_{max}^{DC} \tag{16}$$

(10), being the objective function, states that total costs for electricity supply shall be minimized. Costs arise from producing electricity on the one hand and costs related to transmission grid extensions on the other (note that for the sake of simplicity the expansion of generation capacity is not included at this stage. This condition can easily be relaxed, as done in the large-scale application presented in Section IV). The equilibrium condition (11) ensures that supply equals demand in each market region i at every instant in time t. Electricity can be supplied either by generation in the local market or by imports from other markets. Trades can be settled via AC or DC grid infrastructures, as stated in (12). Once trade flows are set, the resulting physical flows can be calculated: For the AC grid, we use the methodology based on PTDFs as introduced in section II-A and recaptured in (13). For the DC grid, trades directly translate into physical flows (14). The last two Equations (15) and (16) restrict the resulting flows to the line capacities $\boldsymbol{P_{max}}$ that are currently installed. Line capacities in turn are subject to optimization.

As shown in section II-A, the PTDF matrix depends on the physical characteristics of the AC grid, especially on line reactances. When AC grid capacities change, the PTDF matrix will also change. Thus, whenever the optimal solution includes increasing line capacities, the underlying PTDF matrix that was used to deduce the optimum is no longer a valid one for the resulting system. Consequently, a new PTDF matrix is calculated based on the new grid infrastructure, and updated within the above optimization problem. The problem therefore has to be solved iteratively while updating the PTDF matrix every time the market model has found an optimal solution. A schematic representation of the resulting process is shown in Figure 1.

Note that an alternative approach to the process described in Figure 1 would be the calculation of the PTDF matrix directly in the market model according to (9). However, as the elements of $\boldsymbol{PTDF}$ depend on the line capacities

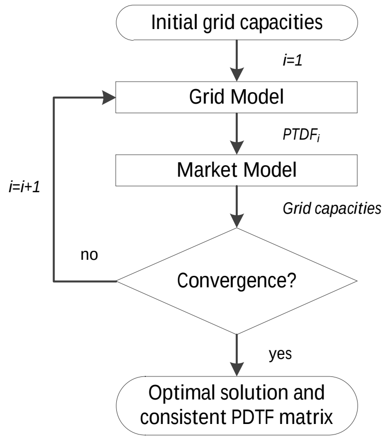

Fig. 1. Schematic representation of the iterative process

$\boldsymbol{P}^{AC}_{max}$, this would make (13) non-linear and the optimization problem difficult to solve, especially in large-scale applications. In fact, there are very effective methods for solving linear programming problems, such as the Simplex algorithm (see e.g. [11], [12]), whereas algorithms for non-linear optimization problems are either inefficient or only find a local instead of the global optimum (see e.g. [13]).

III. Three node network

In this section the methodology developed in Section II shall be applied to a simple example. A three node network was chosen as this is the easiest setting for which loop flows play a role.

With a given demand as well as fixed available generation capacities and costs, the cost-optimal solution for a full electricity supply shall be found that potentially involves transmission grid extensions.

A. Setting of the three node network example

The setting of the three node network considered in this part of the analysis is shown in Figure 2.

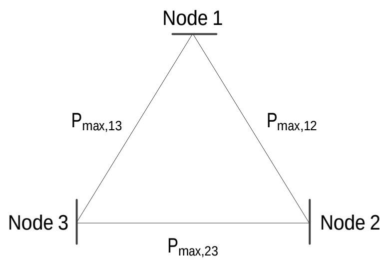

Fig. 2. Three-node network considered in this section

With (9), the transaction-based PTDF matrix for this network can be calculated as in the following equation. Each entry of the matrix is labeled with the corresponding transaction $T_{i,j}$ and impact on line $L_{i,j}$ in order to facilitate reading.

$$\boldsymbol{PTDF} = \frac{1}{x_{12}+x_{13}+x_{23}} \begin{array}{c} \\ L_{13} \\ L_{12} \\ L_{23} \end{array} \begin{array}{c} \begin{array}{ccc} T_{13} & T_{23} & T_{12} \end{array} \\ \left[\begin{array}{ccc} x_{12}+x_{23} & x_{23} & x_{12} \\ x_{13} & -x_{23} & x_{13}+x_{23} \\ x_{13} & x_{13}+x_{12} & -x_{12} \end{array}\right] \end{array} \tag{17}$$

As in [6], we assume additional line capacity is added in parallel, such that the reactances' dependency on line capacity takes the following form:

$$x_{ij} = \frac{x_{ij,0}}{P_{max,ij}/P_{ij,0}}. \tag{18}$$

Note that the algorithm needs a starting point for the iteration. Starting from an initial guess for line capacities and corresponding line reactances the algorithm iteratively searches for optimal grid capacities while updating line reactances according to (18).

We assume different generation and load levels at each node that are exogenous and constant. We then consider a 10 year planning horizon for which the grid shall be optimized. Generation costs at node 1 and 2 are 20 Eur/MWh or 1.752 Mio. Eur per MW supplied for 10 years, and 15 Eur/MWh or 1.314 Mio. Eur/(MW*10a) at node 3. Costs for grid upgrades amount to 1000 Eur per MW and km, with distances of 300 km between all nodes. Furthermore we assume that for security reasons a minimum of 50 MW shall be built on each line which adds an additional restriction to the optimization problem formulated in Equations 10 to 16.

Table I summarizes load level P_{load}, available generation capacity P_{gen} and generation costs C_{gen} at each of the three nodes, as well as the costs for grid upgrades.

TABLE I
Assumptions for the three node network example

Parameter	Unit	Node 1	Node 2	Node 3
P_{load}	MW	800	300	200
P_{gen}	MW	300	300	800
C_{gen}	Mio.Eur/(MW*10a)	1.752	1.752	1.314
		Line 1-2	**Line 1-3**	**Line 2-3**
C^{AC}_{tran}	Mio.Eur/MW	0.3	0.3	0.3
C^{DC}_{tran}	Mio.Eur/MW	1.5	1.5	1.5

B. Results of the three node network example

Based on the assumptions listed in the previous Section we run the model as it was presented in Section II in order to determine necessary grid extensions when all three nodes shall be connected through an AC and/or DC transmission grid. The results are presented in Figures 3 to 5 that capture all endogenous system properties that are subject to change when running the iterative simulation. Noticeably, in this example DC grid extensions are not part of the optimal solution due to higher investment costs compared to AC transmission grids.

For the initial guess all line capacities and reactances were set to 100 MW and 1 Ohm, respectively. Consequently, a power transfer of x from Node A to B results in power flows of 2/3*x on line A-B and 1/3*x on lines A-C and C-B. Node 1 is lacking 500 MW of power generation that needs to be imported from outside. Due to lower generation costs and availability, the missing 500 MW are supplied by Node 3. Transmission lines are extended such that these 500 MW can be transported from Node 3 to Node 1, hence 2/3*500 MW flow on line 1-3 and 1/3*500MW via node 2 (i.e. on line 1-2 and line 2-3). Furthermore, the optimal solution includes the usage of the full capacity available at lower costs in Node 3.

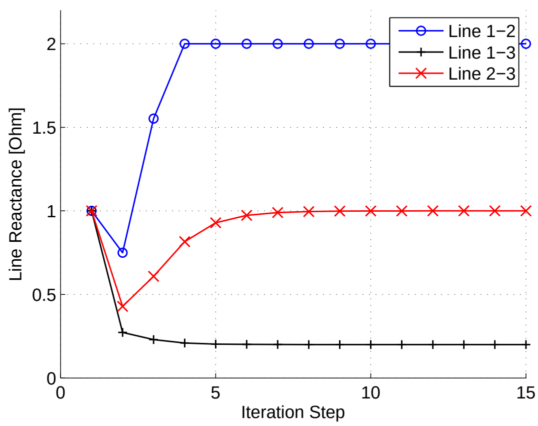

Fig. 3. Development of transmission line reactances during the iteration

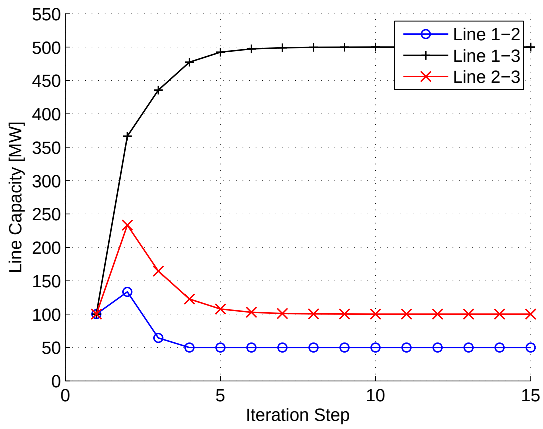

Fig. 4. Development of transmission line capacities during the iteration

The inherent advantage is that the flows on line 1-2 resulting from the trade between nodes 3 and 2 counteract the flow from 3 to 1 via 2, thus leading to a situation where less grid extensions are needed on this particular line. Necessary grid extensions in iteration step 1 then amount to 133 1/3 MW on line 1-2, 366 2/3 MW on line 1-3 and 233 1/3 on line 2-3. Total costs sum up to 2.1472 Mio. Euros.

In the next iteration step, line reactances are updated according to (18) and previously optimized line capacities, thus changing the PTDFs, power flows and optimized line capacities. As can be seen in Figure 4, necessary upgrades on line 1-3 further increase. This is caused by the following sequence of events: capacity extension on this line is highest, and therefore, line reactance decreases the furthest. As more power flows on lines with low reactance, larger transmission capacities are needed on line 1-3 in order to handle the increasing power flows. Following the same logic, necessary transmission capacities on lines 1-2 and 2-3 decrease. Noticeably, as only one line is affected by increasing transmission capacities whereas the capacities of two lines are reduced, total system costs are lowered during the iteration as can be observed in Figure 5. The reactance of line 1-2 increases sharply in the third iteration step since only moderate capacity upgrades were found to be cost-optimal. During the next iterations, capacity of line 1-2 further

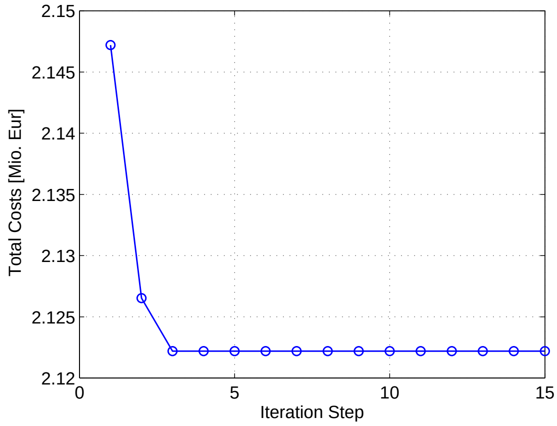

Fig. 5. Development of total system costs during the iteration

decreases and rapidly approaches the minimum extension capacity of 50 MW with corresponding line reactance of 2 Ohm.

During the iterative process, all endogenous system properties reach stable levels after only a few iteration steps. Optimized line capacities, for instance, change by less than 0.1% after iteration step 10.

IV. Large-scale application

This section presents an outlook on further work that applies the previously developed method to a large-scale problem. Specifically, an electricity market model and a power flow model both covering the European power system are coupled via PTDFs in order to find the optimal power system infrastructure development under the prescription of strongly decreasing CO_2 emissions in Europe until 2050 with a specific focus on photovoltaic (PV) power. The following two main questions shall be answered within this framework:

- What does a cost-optimized European power system (both generation and grid) look like in 2030 (medium term) and 2050 (long term)?
- How does an optimized grid extension help to cost-optimally deploy power from PV installations in Europe?

The two models are introduced in Sections IV-B and IV-A, whereas the iteration between them is described in Section IV-C. Please note that the results of the large-scale application are still being finalized and will be published in a separate paper.

A. *Power flow model*

To analyse the power flows in the European transmission network, a detailed model of the high voltage grid is used. This model was developed with DIgSILENT's power system calculation tool PowerFactory and covers all ENTSO-E members. It consists of a total of over 200 nodes, representing generation and load centers within Europe, 450 high voltage AC (HVAC) lines and all the high voltage DC (HVDC) lines within the ENTSO-E area. The grid model is built for AC load flow calculations and thus can be used not just for active power flows, but also to calculate losses within the network, reactive power flows and the necessary compensation to maintain network stability.

As a starting point for the iterations two versions of the grid model were prepared: one representing the European grid as it was in 2011 and another for the predicted state of the network in 2020. For the future projection, it was assumed that all projects in mid-term planning from ENTSO-E's Ten Year Network Development Plan will be built. In total 82GVA of extra capacity was added for new HVAC lines and 13GVA for new HVDC lines between 2011 and 2020.

Whereas in the market model all the load and generation is aggregated for each market region (i.e. assuming a copper plate with no internal power transfers), the grid model consists of multiple nodes per market region.[1] The distribution of demand and generation assets across the nodes within each market region are set using allocation keys, which are based on factors such as population density, siting of heavy industry, location of thermal power plants and the availability of renewable energy resources.

The network and distribution keys were validated by comparing cross-border flows in the model against publicly available data from ENTSO-E, after which the impedances and allocation keys were optimized to ensure good agreement across several snapshots of the network.

In 2011 and 2020 the majority of DC lines lie between the different synchronous zones of the ENTSO-E area, such as the undersea connection between France and Great Britain. To allow the economic model the choice of extending the HVDC network, an Overlay Network of DC lines was constructed for 2030 and 2050 with DC connections permitted between all neighboring market regions, including those within the same AC network. To take account of the effect of DC transfers on the AC grid, a PTDF for DC transactions was calculated, in addition to the AC PTDF for transactions inside the AC network. This DC PTDF linearizes the effect of DC transfers on the AC network, capturing for example the power flows to and from the DC connection points.

An additional challenge was presented by the fact that each market region spans several nodes within the grid model. To accurately capture the flows between the nodes inside each region, which change depending on the dispatch of generation technologies at any given time, the node allocation keys ($\boldsymbol{K}$) were directly incorporated into the PTDF. In this way the nodal power balances within the model can be determined for any dispatch situation, with the power flows then following directly from the usual PTDF. Thus (13) is reformulated as follows:

$$\boldsymbol{P}^{AC} = \boldsymbol{PTDF} \cdot (\boldsymbol{K}^{D} \cdot \boldsymbol{D} - \boldsymbol{K}^{G} \cdot \boldsymbol{G} - \boldsymbol{K}^{DC} \cdot \boldsymbol{T}^{DC}) \quad (19)$$

B. Electricity market model

The market model used in this analysis is an extended version of the long term investment and dispatch model for conventional, renewable, storage and transmission technologies as presented in [14]. It covers 29 countries (EU27 plus Norway and Switzerland) at an aggregated level (i.e. 18 market regions). [2]

The model determines possible paths of how the installed capacities will develop and how they are operated until 2050 under different assumptions, assuming that the European markets will achieve the cost-minimizing mix of different technologies - a market result that is set in full competition. The objective of the model is thus to minimize accumulated discounted total system costs while being subject to several techno-economic restrictions, such as the hourly matching of supply and demand, fuel availabilities or potential space for renewable energies, and politically implied restrictions, such as an EU-wide CO_2 emission reduction target or limited nuclear power deployment. The dispatch is calculated for eight typical days per year on an hourly basis (scaled to 8760 hours), representing variations in electricity demand as well as in solar and wind resources along with their multivariate interdependencies. Extreme events that particularly stress the power system, e.g. periods of low wind and high demand, are also covered. To account for local weather conditions, the model considers several wind and solar power regions (subregions) within market regions based on hourly meteorological wind speed and solar radiation data [15]. For the study presented in this paper, the grid optimization has been implemented in the electricity market model as in equations (10) to (16) and (19).

An important assumption concerns the congestion management, i.e. the restriction of electricity transactions between market regions: As opposed to NTC-based market coupling which is still the predominant method for congestion management in the European power system, our calculations are all based on a flow-based market coupling regime. The reason for this is two-fold:

- Flow-based market coupling is implemented in the market model in order to optimize thermal limits of the transmission grid directly and without having to calculate Net Transfer Capacities (NTC) every time the grid infrastructure is changed. This results in a clearly defined interface between the market model and the power flow model, namely the PTDF matrix.
- Previous studies have shown that flow-based market coupling increases market efficiency, and should thus be chosen in order to determine the cost-efficient electricity supply while optimizing social welfare. [3]

C. Iteration between the models

Years of reference included in the analysis are 2011, 2020, 2030, 2050. As described in Section II, the interface for the power system optimization is the PTDF matrix. It is initially calculated from the flow model for the years 2011 and 2020 for which the grid infrastructure is not optimized; we argue that for the year 2020 optimized grid extensions would not be realistic within this timeframe, due to long planning and

[1] Note that this could be overcome by simulating a nodal pricing regime where each node of the transmission grid is its own market region. However, this would call for a market model that is even more complex than the one that is currently used and could thus not be solved in a reasonable time.

[2] The aggregation was done due to very long computational times

[3] For a general discussion of flow-based transmission rights and congestion management see [16]. Analyses of different congestion management regimes in the European power system and possible increases in market efficiency were published in [17], [18], [19], [20], [21]. Practical feasibility of the concept is currently proven in the Central Western European (CWE) Region, as discussed in [22].

permission procedures of such projects. For the year 2011, the model represents current line capacities while for 2020 a number of mid-term grid extensions are included as reported in the Ten Year Network Development Plan (TYNDP) [23].

For later years, however, optimal grid extensions are allowed leading to variations in line capacities. These extensions alter the impedances within the network model, which in turn change the PTDF. Since the way the PTDF changes is non-linear, it cannot be incorporated directly into the linear optimization problem, so instead the PTDF is updated iteratively until it converges on the optimal consistent solution. As a starting point for the 2030 and 2050 networks, the 2020 PTDF is used. Note that in the electricity market model, generation capacities are optimized starting from 2011.

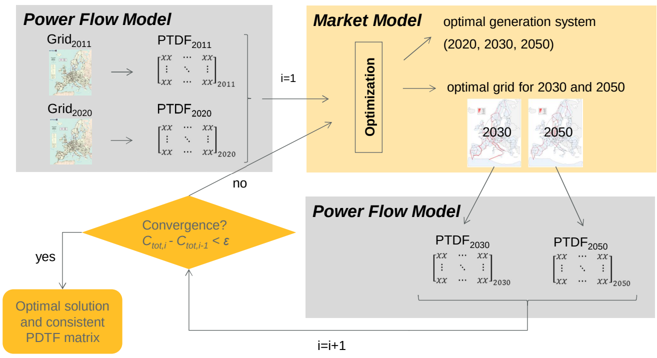

Fig. 6. Iteration between the market and the power flow model of the European power system

V. Conclusions

Joint optimization of generation and transmission is an extremely important yet difficult task, mainly due to the fact that different characteristics and rules apply to commercial and physical exchanges of electricity in meshed networks. In this paper a method is developed based on an iterative PTDF calculation that is suitable for determining the cost-optimal extension of large-scale power systems - such as the European interconnected network - including generation as well as grid facilities. An interface is implemented based on PTDF matrices that couples market and grid models and combines the inherent advantages of both model types. Specifically, the algorithm is formulated as a linear optimization problem that can be solved efficiently. It is tested for a simple three node network where it is found to be stable and convergent.

The method is currently implemented for a large-scale case study to find an optimal power system infrastructure development under the prescription of large shares of photovoltaics in Europe until 2050 by using a linear European Electricity Market Model and a European Transmission Network Model. Within this framework, it is analysed how the cost-optimal European power system develops until 2030 (medium term) and 2050 (long term), and how enhanced grid extensions help to cost-optimally deploy power from photovoltaic (PV) installations in Europe. First results indicate that - as expected - without grid extensions electricity needs to be supplied more locally. However, the costs savings enabled by optimal grid extensions is not as pronounced due to strongly decreasing costs of renewable energy technologies that diminish the importance of local resource quality (namely wind and solar radiation). Furthermore, it can be observed that large-scale PV deployment calls for storage devices in order to balance out the diurnal variations.

The approach presented in this paper could be further developed in various directions, specifically with respect to the market model that could be formulated as a different class of optimization problem. The most interesting ones would be:

- A mixed integer problem in order to formulate that only multiples of available line configurations can be added.
- A non-linear optimization problem in order to avoid the iteration.
- Further spatial disaggregation towards a nodal-pricing regime in order to overcome the difficulty of having an unequal number of nodes in the power flow and market model.

Furthermore, it could be analysed numerically how gains in social welfare can be created when switching from NTC to flow-based market coupling all over Europe.

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